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AAG-003-001616 Seat No. ________

B. Sc. (Sem.-VI) (C.B.C.S.) Examination

April / May – 2016

Mathematics : BSMT-601(A)

(Graph Theory & Complex Analysis-2)

(New Course)

Faculty Code : 003

Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Section-A carries 20 marks of MCQs and Section-B carries 50 marks.

(3) Answer all MCQs in answer book only.

1 Choose correct option for the following M.C.Qs. **20**

(1) The maximum number of edges in a simple connected graph with 10 vertices are ________.

(A) 40 (B) 45

(C) 50 (D) None of these

(2) A null graph of two or more vertices is always ________.

(A) connected (B) disconnected

(C) 0-regular graph (D) both (B) and (C)

(3) A minimally connected graph G is always a ________.

(A) Tree (B) Circuit

(C) Eulur Graph (D) None of these

(4) The number of vertices of odd degree in a graph is always ________.

(A) odd (B) prime

(C) even (D) None of these

(5) A vertex in a graph G having degree one is called ________ vertex.

(A) Isolated (B) even

(C) pendant (D) None of these

(6) In a connected graph G any minimal set of edges containing at least one branch of every spanning tree is called __________.

(A) Parallel edges (B) Cut-set

(C) Circuit (D) None of these

(7) A Hamiltonion circuit in a graph G of order n consists of exactly __________ edges.

(A) $n - 1$ (B) n

(C) $n + 1$ (D) None of these

(8) A tree in which there is exactly one vertex of degree two and each of the remaining vertices is of degree one or three is called __________.

(A) spanning tree (B) non-rooted tree

(C) binary tree (D) None of these

(9) Regions of a connected planner graph with 4-vertices and 6-edges are __________.

(A) 1 (B) 4

(C) 2 (D) None of these

(10) The dimension of circuit subspace W_T is equals __________ of the graph.

(A) rank (B) nullity

(C) order (D) None of these

(11) A polynomial of degree n has pole of order n at __________.

(A) infinity (B) zero

(C) anywhere (D) None of these

(12) The pole of first order is known as __________.

(A) complex pole (B) simple pole

(C) singularity (D) None of these

(13) The radius of convergence of power series $\sum \frac{z^n}{n!}$ is __________.

(A) $z = 0$ (B) $z = 1$

(C) $z = \infty$ (D) None of these

(14) To integrate $\int_0^\infty \frac{1}{1+x^2} dx$ we will use a contour __________.

(A) real axis and unit circle

(B) real axis and lower half of |z| = R

(C) real axis and upper half of |z| = R

(D) None of these

(15) The function $f(z) = \cos z$ is __________.

(A) analytic-everywhere

(B) singularities at $z = \frac{n\pi}{2}$

(C) singularities at $z = \frac{(n+1)\pi}{2}$

(D) None of these

(16) The value of the integral $\oint_C \frac{e^z}{z-2} dz$, where C : |z| = 3 is __________.

(A) $2\pi i e^2$ (B) $2\pi i$

(C) e^2 (D) None of these

(17) The singularity of the function $f(z) = \frac{z - \sin z}{z^2}$ is __________.

(A) z = 0 (B) z = 2

(C) z = –2 (D) None of these

(18) The fixed points of the mapping $w=\dfrac{5z+4}{z+5}$ are

(A) 2, 2 (B) 2, –2

(C) –2, –2 (D) None of these

(19) The invariant point of the transformation $w=\dfrac{z-1}{z+1}$ are __________.

(A) $z=\pm 1$ (B) $z=\pm i$

(C) $z=\pm \dfrac{i}{2}$ (D) None of these

(20) If the function f has a pole of order m at $z=a$ and $g(z)=(z-a)^m \cdot f(z)$ then

(A) $\operatorname{Res}(f;a)=\dfrac{g^{m-1}(a)}{(m-1)!}$

(B) $\operatorname{Res}(f;a)=\dfrac{g^{m}(a)}{(m-1)!}$

(C) $\operatorname{Res}(f;a)=g^{m-1}(a)$

(D) None of these

2 (a) Attempt any **three** : 6

(i) Define : Regular Graph, Complete Graph.

(ii) What is size of a K-regular (n, e) graph ?

(iii) How many vertices are needed to construct a graph with 6-edges in which each vertex is of degree two ?

(iv) Define Rank and nullity of a graph. What are the rank and nullity of complete graph K_n ?

(v) Define : Path Matrix with example.

(vi) Define : Proper Vertex Colouring and Chromatic Number of a Graph.

(b) Attempt any **three** : **9**

(i) State and prove first theorem of graph theory.

(ii) Prove that complete graph Kn has always a Hamiltonian Circuit.

(iii) Prove that a tree with n-vertices has n-1 edges.

(iv) Show that the number of vertices in a Binary tree is always odd.

(v) Define : Incidence Matrix and state its properties.

(vi) Prove that every tree with two or more vertices is 2-chromatic.

(c) Attempt any **two** : **10**

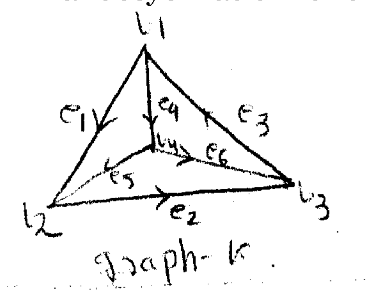
(i) State and prove the necessary and sufficient condition for a graph to be an Euler Graph.

(ii) If G is a graph with n-vertices, e-edges, f-faces and k-components, then show that n-e+f = K+1.

(iii) In usual notation prove that $(W_G, \oplus)$ is an abelian group.

(iv) Prove that rank of an incidence matrix A(G) of a connected graph G, with n-vertices is n-1.

(v) Find minimal decyclization for the following graph.



3 (a) Attempt any **three** : **6**

(i) Find region of convergence and radius of convergence for the series $\sum_{n=1}^{\infty} \frac{z^n}{3^n+1}$

(ii) Prove that

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{Z^6}{6!} + \ldots\ldots$$

(iii) Define pole of a complex function.

(iv) Find residues of $f(z)=\dfrac{z+2}{(z-1)(z-2)}$ at simple poles.

(v) Find critical point of $w=\dfrac{z-1}{z+1}$

(vi) Find fixed points of the transformation

$$w=\frac{-2+(2+i)z}{i+z}$$

(b) Attempt any **three** : **6**

(i) Expand $\dfrac{1}{(z-1)(z-2)}$ in Laurent's series for

$|<|z|<2$.

(ii) If Z_0 is the m^{th} order pole of complex function $f(z)$ then prove that

$$\operatorname{Res}(f(z), z_0)=\frac{\phi^{m-1}(z_0)}{(m-1)!} \text{ where } \phi(z)=(z-z_0)^m f(z)$$

(iii) Evaluate $\displaystyle\int_c \frac{2z+3}{z(z-1)}dz$, $C:|z|=2$

(iv) Prove that $\displaystyle\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}=\frac{2\pi}{\sqrt{3}}$

(v) Show that the composition of two bilinear maps is again a bilinear map.

(vi) Discuss the critical points of bilinear map $w=\dfrac{az+b}{c2+d}$,

$ad-bc\neq 0$

(c) Attempt any **two** : **10**

(i) State and prove Taylor's infinite series of an analystic function $f(z)$

(ii) Expand $f(z)=\dfrac{1}{z^2 \sin h^2}$ in Laurent's series for

$z_0=0$ hence deduce that $\int_c \dfrac{1}{\sin h_2}\,dz = 2\pi i$ and

$\int_c \dfrac{1}{z^2 \sin h_2}\,dz = \dfrac{\pi}{3}i$

(iii) Using residue theorem prove that

$$\int_0^\infty \frac{dx}{x^4+1}=\frac{\pi}{2\sqrt{2}}$$

(iv) Prove that $\displaystyle\int_0^\infty \frac{dx}{\left(x^2+a^2\right)^{n+1}}=\frac{\pi(2n!)}{(2a)^{2n+1}(n!)^2}$

where a>0.

(v) Obtain the transformation of real line (axis) of z-plane, under the mapping $w=\dfrac{2+iz}{i+4z}$